The Bella Curve and load bearing capacity towards wind

Rune Brincker, august 2026.

Abstract

In this paper we are considering linear elasticity theory to model the maximum horizontal force on a building that can only carry compression stresses from the gravity load, such as masonry and unreinforced concrete buildings. To solve this problem, we define two simple dimensionless geometrical quantities and define a corresponding plot between the two quantities that we denote the Bella Curve. This curve can be used to provide an estimate of the moment arm of the normal force to withstand the above mentioned maximum horizontal force so that we still have compression stress in all load bearing walls. We illustrate the principle on classical beam sections and then also demonstrate how the principle can be applied to provide an estimate of safety towards wind for the Danish high rise building from Bellahøj and for the 670-year-old Giotto Bell Tower in Italy.

1 Introduction

The background of this paper is a public debate in the Danish society in the spring of 2026 about whether or not it is reasonable to tear down 10 of the famous Bellahøj buildings that was built in the mid nineteen fifties in the north of Copenhagen.

However, the focus of this paper is not on this public debate, but solely on the aim to provide a simple way to estimate the load bearing capacity towards wind of any building with limited capacity to carry tensile stresses so that we need to make sure that we have compression stress in all load bearing walls.

We will also consider the wind load and the overall safety in this paper, but it has a secondary priority and is only present to illustrate the importance of estimating the load bearing capacity secured by the gravity load of a building.

We will consider the building as a vertical beam loaded by the vertical normal force N from the gravity load and the horizontal force F representing the force from the wind, see Figure 1.

The vertical force N is considered to be constant, but the horizontal force F can be varying from zero up to the maximum load that we will denote $F_{\max}$. When the horizontal force $F=0$ the moment from the horizontal force is zero, and we just have the pressure stress from the normal force that is uniformly distributed assuming a symmetrical section with area A, so that the compression stress is equal to N/A. See the bottom plot to the left of the coordinate system plot in Figure 1.

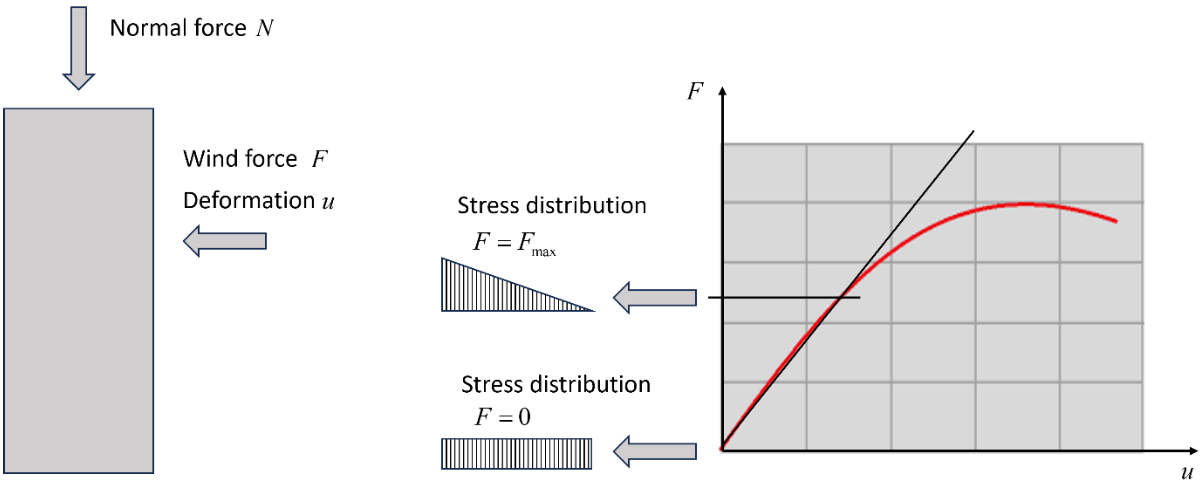

Figure 1. *To the left the considered building loaded by the compression normal force N due to the gravity load and the horizontal force F. To the right the load deformation curve describing a classical failure curve with initial elastic behavior.*

It is not elementary that the compression stress is equally distributed and equal to N/A, because the distribution of stresses over the different walls in reality depends on how reinforcement in the decks has been designed and the concrete casting process has been carried out.

The distribution of the vertical stresses should in general be considered as a statically indeterminate problem, but since the Danish code, DS 411, [1] that was valid when the Bellahøj buildings were designed and built, states that the elasticity theory can be applied for statically indeterminate problems[1], we will use the equally distributed stress for normal force only assuming that the cross-section of the building is symmetric. More about this issue in the following.

The discussion here is just to introduce the basic idea on the how to deal with the problem of determining the maximum force F_{max}. As illustrated in Figure 1, when the horizontal force F has a higher value than zero, a linear relationship between the force F and the corresponding horizontal deformation u will always be present because all materials are linearly elastic for moderate stress and strain.

When the building is loaded by the horizontal force F acting towards the left in Figure 1, the building is also tilting in this direction, and due to the linear elasticity theory, a linear distribution of the stresses from the moment of the horizontal force is superimposed onto the constant stress from the normal force. As a result, the compression stresses in the right side of the building is reduced and the stresses in the left side of the building is

[1] Elasticity theory can be applied, see page 35 at the top

correspondingly increased until we reach the point where the compression stress at the right side of the building is equal to zero. See the top plot to the left of the coordinate system plot in Figure 1.

We will define the vertical load at this point as the maximum load $F_{\max}$. We could have accepted a little more load either corresponding to possible tensile stress in the concrete, or a small acceptable crack at the button of the building, but we will not do that in order to keep it simple and on the safe side.

We will keep the simple assumption of zero stress for the ultimate state that represents the load bearing capacity:

> *as long as we have moderate compression stress in all the load bearing walls in a building, then we have full safety*

We will not in this paper go into detailed principles of safety, but mainly focus only on the how the elasticity theory can be used in an easy way by introducing two simple geometrical quantities needed to define the Bella Curve

- The area of the transverse load bearing walls of the building
- The eccentricity of the normal force corresponding to the maximum load $F_{\max}$

Both of these two quantities help us to define the Bella Curve so that we can carry out a simple check of security for any building with an approximate symmetrical section.

In the following we will define the two geometrical quantities mentioned above and consider the formulation of the Bella Curve. Then later we will illustrate the use of the Bella Curve on classical thin-walled sections of steel and wood, and finally we will illustrate how to use the Bella Curve to reach an estimate of safety for the both the considered Bellahøj building and a of one of the oldest and highest masonry buildings in Europe.

2 Generic section and the Bella Curve

We are considering a generic double-symmetric cross-section of a beam as shown in Figure 2 where the web with the thickness t_w can be divided up into n_w number of webs with the web thickness t_w / n_w without changing the normal stresses at the section.

Dividing the web into two on each side of the section we form a rectangular hollow section (RHS) profile, and dividing the web into more webs, we can modify the section to an arbitrary wall geometry of nearly any building.

The idea of the Bella Curve is to make a smooth transition from the case of an extremely thin-walled section to the simple rectangular section by increasing the thickness of the web from zero to the beam width b.

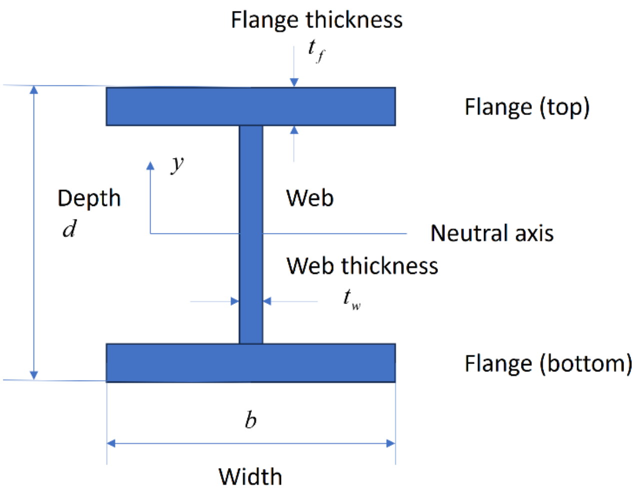

Figure 2. *Symmetric generic cross-section where the web can be divided up into n_w number of webs with the web thickness t_w / n_w without changing the normal stresses at the section.*

In order to consider the Bella Curve, we assume that the material is linear elastic so that we can use the general Navier Equation, see [2][2] for the normal stresses at the section

(1) $$\sigma = \frac{N}{A} + \frac{M}{I} y = \sigma_N + \sigma_M$$

where A is the section area, I is the inertial moment, N is the normal compression force at the middle of the section and M is the bending moment acting around the neutral axis of the section. A more internationally reference on the subject is given by [3][3].

Now we move the vertical force N away from the neutral axis so that the moment arm a of the normal force times the normal force becomes equal to the moment

(2) $$M = Na$$

First, we will consider the case where the moment arm is just so large that the minimum compression stress just becomes equal to zero, that is when

(3) $$\frac{N}{A} = \frac{Na}{I}\frac{d}{2} \quad \Rightarrow \quad \frac{1}{A} = \frac{a}{S}$$

where S if the Elastic Section Modulus for the symmetric section

(4) $$S = \frac{I}{d/2}$$

The maximum moment arm where we have compression in all the section is then given by

[2] See chapter 5 Eq (5.40)
[3] Ribbeler is not referring to Navier, but you can look for combined loadings.

(5) $$a = S / A$$

We can then define the dimensionless eccentricity, that we can also denote the eccentricity degree

(6) $$\frac{a}{d} = \frac{S}{Ad}$$

We will now consider the two well-known extreme cases, first where we are considering the simple rectangular section, where the maximum moment arm is minimum, and then the case of extremely thin-walled section where the moment arm is maximum.

Considering the first case of a rectangular beam section we have $S = bd^2/6$, $A = bd$ and we get

(7) $$\frac{a}{d} = \frac{bd^2/6}{bd^2} = 1/6$$

so that the minimum eccentricity degree is 1/6 and the moment arm has the minimum value

(8) $$a_{\min} = d/6$$

The eccentricity degree of 1/6 for the rectangular section is a well-know result, normally expressed as we have “only compression stress” when the normal force is acting in the middle third part of the section.

Considering the extremely thin-walled section we have the estimate of the inertial moment that is exact for in the limit $t_f \to 0$

(9) $$I = \frac{1}{12} t_w (d - 2t_f)^3 + 2t_f b((d - t_f)/2)^2$$

And using Eq. (4) and obtaining the corresponding area we have

(10) $$A = 2t_f b + t_w (d - 2t_f)$$

Now considering limit $t_w \to 0$ we get

(11) $$S = \frac{2t_f b((d - t_f)/2)^2}{d/2}$$

and then for the limit $t_f \to 0$ we are then considering an extremely thin-walled section, and we get

(12) $$S = \frac{2t_f b(d/2)^2}{d/2} = 2t_f b(d/2)$$

So that using $A = 2t_f b$ we obtain

(13) $$\frac{a}{d} = \frac{S}{Ad} = \frac{2t_f b(d/2)}{2t_f bd} = 1/2$$

This is also a well-known result, that a thin-walled beam where the area web is so small that it can be neglected, and the flanges are thin, we have a moment arm has the maximum value

(14) $$a = d/2$$

Now we will be considering all the cases between these two extremes providing us with the data to plot the Bella Curve.

The idea is to make the dimensionless plot between the eccentricity degree defined in Eq. (6) and the dimensionless area also denoted the area degree A/A_0, where $A_0 = bd$, i.e. equal to the area of the rectangular section.

Since the Bella Curve is a dimensionless solution to how stresses distribute in an elastic beam due to normal force and bending moment, and that the distribution of stress is not dependent on elasticity as long as the section is homogeneous, the Bella Curve is true for any double symmetric section consisting of two flanges and a number of webs in between them.

In Figure 3 we have shown the results of plotting the dimensionless Bella Curve, and we can see that we do not have only one Bella curve, but a continuum of Bella Curves depending on the relative thickness of the flange.

In Figure 3 we have shown three Bella Curves covering most practical cases, that its relative flange thickness of 1, 2 and 5 %. We see that they are all increasing the eccentricity towards 0.5 when decreasing the relative web-area towards zero.

In the following we will therefore take a look at very different beams, first simple small beam often used on building construction, and then we will take a look at a few buildings, one of the 10 Bellahøj buildings that are considered to be teared down, and then an old masonry tower from Italy.

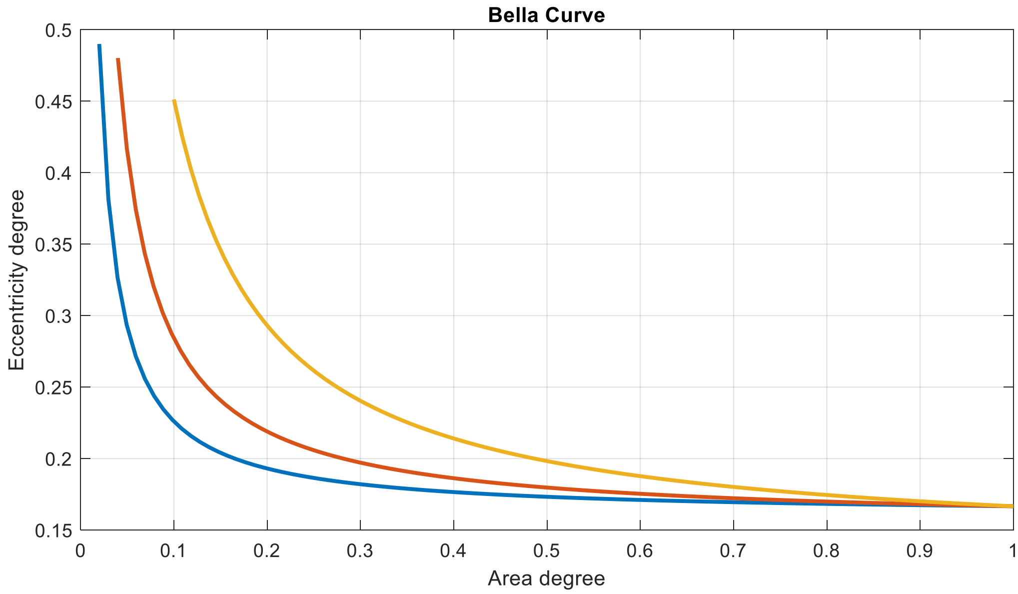

Figure 3. *Eccentricity degree defined in Eq. (6) as a function of the area degree* A / A_0 *. From the bottom the three curves represent cases where the flange thickness is 1, 2 and 5 % of the beam depth.*

3 Classical beam sections

Since the Bella Curve is a dimensionless description of how the stresses from normal force and bending distribute in a beam with a double symmetric section, we will consider a few classical beam sections just to illustrate the general application value of the Bella Curve principle.

Table 1 shows a few beams of this kind. Most of the beams in Table 1 are classical steel beams used often used in civil and mechanical engineering, and they can all be found in for instance Teknisk Ståbi, see [3] or in open access links like [4].

When we plot the steel beams into the Bella Curve as shown in Figure 4 it fundamentally illustrates that for this kind of beams the dimensionless quantities that we denote the eccentricity degree is lying in the interval around or a little below 0.35 to 0.40. This can of course be used to find the maximum horizontal force in the case the steel beam is standing vertical and supported on an elastic foundation where we need to know the maximum horizontal force that corresponds to zero stress in in the side where the compression stresses are reduced due to the horizontal force.

However, for a steel beam this investigation might not make any practical sense because the steel material can carry significant tensile stresses, so that accepting a higher horizontal force than the force that we estimate in section 2 defined as the maximum force is not a problem at all. Actually, if the horizontal force exceeds the earlier defined maximum force, the side of the beam that is lifted up will just become stress free and in the moment arm will then be equal to $a = d/2$ as for the very think walled case shown in Eq. (14).

This just explains and illustrates why we consider the Bella Curve as important for structures that is made of a material that can easily carry compression stress, but cannot so easily carry tensile stresses like unreinforced concrete and masonry. The more thind walled the section is, the bigger the moment arm, and the more vertical load can be carried just by the gravity force.

However, we can also in the Bella Curve plot see the degree of flange thickness, like for instance the RHS profile has a little higher flange thickness than 2 % of the depth, and the HEA has a flange thickness that is close to 5 % of the depth.

The wooden box that is often used in buildings as facade and deck elements is different from all the steel boxes due to the fact that is has a significant larger web area but around the same dimensionless flange thickness as the HEB profile that is also evident from Table 1. We can say the same about the wooden box as we said about the steel profiles, the wood can carry tensile stresses and therefore the definition about the maximum force made in the introduction is not relevant for the wood material.

Table 1. *Typical beam sections used in Danish building technology, all dimensions in mm.*

Section type	**Depth**	**Width**	**Web thickness**	**Flange thickness**
IPE	300	150	7,1	10,7
HEA	290	300	8,5	14,0
HEB	300	300	11,0	19,0
RHS	300	200	8,0	8,0
Wooden Box (WB)	300	600	100	19

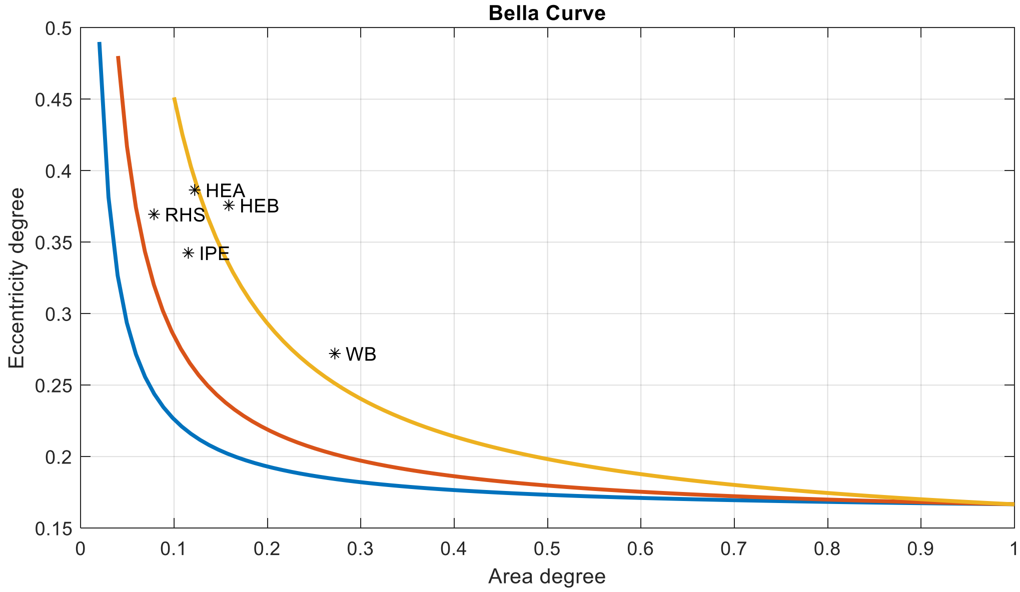

Figure 4. *The Bella Curves at shown in figure 2 plotted together with the classical beam sections shown in Table 1.*

4 Two building cases

The most important structure considered in this paper is a building from the Bellahøj high-rise buildings, or a smaller part of the Bellahøj neighborhood denoted Bellahøj I and II also denoted SAB I and SAB II that constitute a central part of the Bellahøj complex.

Bellahøj I and II was built between 1951 and 1958, and the housing complex has a little less than 500 apartments situated on Copenhagen's highest hill. The project marked a historic shift in Danish residential construction by introducing industrialized methods for concrete structures.

The entire development was strongly influenced by the modernist visions of the Swiss-French architect Le Corbusier regarding "houses in the park". The goal was to secure free green spaces by building upwards.

Bellahøj marked the transition from traditional masonry construction to pre-fabricated concrete construction, and the buildings in Bellahøj I and II were among the first in Denmark to be built using concrete elements based on modern principles.

Although Bellahøj I and II constitute a single housing complex in Bellahøj South, the buildings were constructed in two separate phases. Bellahøj I was constructed in 1951–1955 using slip forming for the facade walls as well as for the rest of the walls of the buildings, whereas SAB II was built using prefabricated facade elements in 1955–1958.

SAB II also introduced greater geometric variety. For instance, building 6 in SAB II stands out by being constructed as a building with twice the length of most of the other buildings in SAB II, and this is the building that we will consider here. The reason is that it has the most difficult and detailed geometry of its beam section and as such illustrates the simplicity of using the Bella Curve to reach an estimate of the maximum horizontal force as mentioned in the introduction.

The main problem that we are discussing in relation to the Bellahøj II buildings is if the facade elements can be considered as being cast together with the rest of the structure in such a way that we can consider them being a part of a monolithic structure. This is necessary in order to assume that the facade element is taking part in carrying the gravity load of the building and as such also taking part in carrying the horizontal load according to the elasticity theory as described in section 2.

There have been two reviews of the owner's technical investigations that was leading to the conclusions that the buildings were threatened by collapse due to wind forces and therefore should be teared down in order to secure safety for the people living in the buildings.

The owners came to that conclusion applying current standards and assuming that the facade element in SAB II buildings could not participate in carrying any loads.

However, important remarks from the two reviews are discussed in appendix A. The first review from 2021 indicates that the facade element might actually be able to carry vertical loads, and the second review from 2023 point to the fact that the original safety calculation made in the mid fifties were based on linear elasticity theory using the full wall geometry. Both of these remarks indicate that the facade elements of the SAB II buildings could be considered as load bearing walls.

This leads to the considerations in appendix B where we are considering the geometry of the total section of the considered building and especially the estimation of the thickness of the facade elements. This is important to achieve a reasonable input to the Bella Curve where we have already illustrated that the flange thickness is essential.

In Table 2 we have provided the key geometry for the considered Bellahøj building, that has a length of 28 meters and a depth of 10 meters measured from corner to corner. Using the considerations from Appendix B we have assumed two values for the effective flange thickness as 7 and 9 cm, and two values for the web area, one assuming 9 transverse walls (total thickness 1.35 m) and one assuming 6 transverse walls (total thickness 0.90 m). This defines the four combinations a shown in Table 2 that are plotted into the Bella Curve plot in Figure 5.

We see that the lowest eccentricity is given by B1 corresponding to the thin effective thickness of 7 cm and only 6 transverse walls. It might seem surprising that a larger web area is not in favor of a larger eccentricity, but that is in fact the case. We read the value for the eccentricity degree to 0.237 that we will use in the final section to evaluation of the safety towards wind.

We will compare the results of the Bellahøj building with an old masonry building, The Giotto's Campanile Bell Tower at Firenze, Italy from 1359. It is one of the many high masonry towers built in the same period and has a height of 85 meters.

The Giotto Tower has a square floor plan, meaning it has the same width in both directions, and the walls are 3.9 meters thick on each side at the base. The tower retains its square shape all the way up to the white travertine top, the walls become slightly thinner the higher up we go, but the tower has a well-defined total weight of 18,600 tons, see [10], that we will reduce to 17,000 tons to ensure a lower bound value.

We see that the tower point in Figure 5 is totally to the right of the Bella Curve which is typical for the old masonry towers built in the same period, and we read the eccentricity value of 0.194 that we will use together with the weight in the next sections to calculate the maximum horizontal wind force on the structure.

Table 2. *Geometry of the considered building cases, all dimensions in m.*

Section type	Depth	Width	Web thickness	Flange thickness
Bellahøj B1	10	28	1.35	0.07
Bellahøj B2	10	28	1.35	0.09
Bellahøj B3	10	28	0.90	0.07
Bellahøj B4	10	28	0.90	0.09
Giotto Campanile	14.5	14.5	7.8	3.9

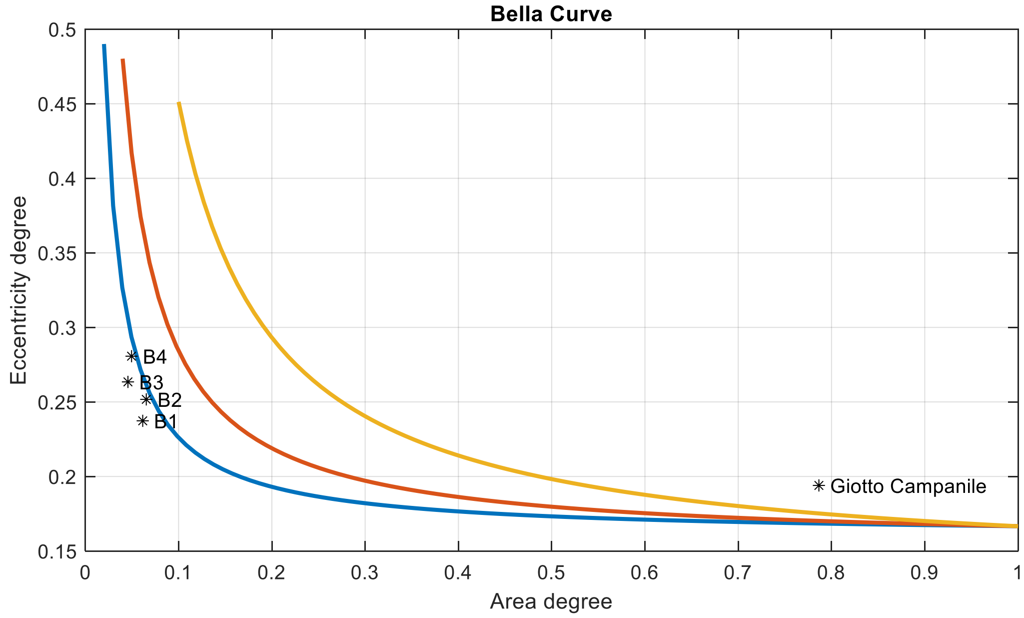

Figure 5. *The Bella Curves as shown in figure 2 plotted together with the two considered buildings shown in Table 3.*

5 Safety towards wind

In this section we will use the Bella Curve to estimate a lower bound for the maximum wind force $F_{\max}$ on the Bellahøj SAB II, building 6 and a masonry tower The Giotto's Campanile Bell Tower at Firenze, Italy, and then estimate the ratio between the lower bound maximum horizontal force and the upper bound estimate of the actual peak wind force.

First let us consider the Bellahøj building that is highest and the biggest of the 10 buildings that has been considered to be teared down in the spring of 2026 with a height $h = 40$ m and width $b = 28$ m and has 13 stories including the penthouse on the top.

We will estimate a lower bound value for the load bearing capacity by considered the smallest eccentricity in figure 5 that is $e = 0.237$ giving us a moment arm of 2.37 m. The normal force is estimated as the lower bound weight of the building. Since Bellahøj are using only solid wall and decks, it is considered a heavy concrete building where the total weight per gross m2 is in the range of 1.000 og 1.800 kg, and we will use 1000 kg/m2 as a lower bound value[4].

We will estimate the total gross area using only 12 stories because we do not count the penthouse floor on the top of the building, and we have using the values from Table 2 therefore the area 10 x 28 x 12 = 3360 m2 resulting in a total weight of 3360 tons

[4] If we use the information in Appendix B and take the actual geometry and the lightweight facade into account we reach a similar number.

corresponding to $N = 33,6$ MN. We then have the maximum moment where we still have pressure in all load bearing walls

(15) $$M_{\max} = aN = 2.37 \times 33.6 = 79.6 \text{ MNm}$$

We will use a well-known rule of thumb for positioning the approximating wind load as a single force acting at 60 % of the building's height so that we have the moment equilibrium equation

(16) $$M_{\max} = 0.6 \times hF_{\max}$$

And we can then isolate the maximum wind force

(17) $$F_{\max} = \frac{M_{\max}}{0.6 \times h} = 3,32 \text{ MN}$$

corresponding to 332 tons.

The details about estimating the wind force can be found in Appendix C where we are focusing mainly on the basic physics and only a little on rules and principles from the standards. One of the few things that we are using is the so-called basic wind speed that is set to 24 m/s at 10 m height for nearly the whole country except in a zone along the Jutland west coast where it is increased to 27 m/s, see [12].

In appendix C we are estimating the total wind force F_w according to the so-called Drag Equation

(18) $$F_w = \frac{1}{2} \rho v^2 C_d A_w$$

where ρ is the mass density of the air (1,25 kg/m3), v is the wind speed, C_d is the drag coefficient, and A_w is the area of the building that is exposed to the wind pressure. In this case the area is $A_w = hb = 1120$ m2.

In order to be on the safe side estimating an upper bound for the wind force we are then scaling the 24 m/s at 10 m height to 60 % of the height of the building (24 m). We estimate the static wind speed at this height to 30.3 m/s, and using the Drag Equation above we then estimate the static wind force to $F_{vs} = 83$ tons. Adding the random wind forces we then obtain the peak wind force to $F_{vp} = 95$ tons.

Now let us consider Giotto's Campanile Tower at Firenze that has a quadratic section with $d = b = 14.5$ m, and a total height of $h = 85$ m. The total mass was estimated in the preceding section to 17,000 tons on the safe side, and we find the corresponding results to the Bellahøj building using the eccentricity from figure 5 for the Giotto Campanile Tower that becomes 0.194 giving the moment arm 2.81 m and we obtain $F_{\max} = 9.4$ MN corresponding to 940 tons. For the static wind speed we find $v_s =$ 44.6 m/s and for the peak load we find F_{vp} = 2.46 MN corresponding to 246 tons.

To achieve safety, it is normal to multiply the wind force by a partial coefficient around 1.5 and divide the load bearing capacity by a partial coefficient around 1.3 to make sure that the total safety factor is higher than around 2 (1,5 x 1,3 = 1,95).

The main results for the two buildings are given below in Table 3. As we can see, there is a lot of safety for both the Bellahøj building and the Giotto Campanile Tower, because the safety factor that should just be higher than 2, is for both buildings much higher than 2, for the Bellahøj building the safety factor is 3.5 and 3.8 for the Giotto Campanile Tower.

It means that both building can stay here for hundreds of years and will need nothing more than a basic maintenance that conserve their initial condition.

Table 3. *The safety factor for the basic wind speeds of 24 m/s in 10 m height scaled to 60 % of the building height.*

Building	Weight [ton]	Eccentricity degree	Max wind load [ton]	Peak Wind [m/s]	Peak force [ton]	Safety factor
Bellahøj SABII	3,360	0.237	332	30.3	95	3.5
Giotto's Campanile	17,000	0.194	940	44.6	246	3.8

Discussions

This paper is just a discussion about whether or not it seems to make any sense to consider using linear elasticity theory on the Bellahøj buildings before any decisions about tearing them down is being made.

The two main assumptions behind these considerations are

- That it might be the case that the facade elements are cast together with the rest of the building to form one monolithic structure
- And this means that simple elasticity theory can be used including the facade elements in the wall geometry

We have argued in this paper that these two assumptions seem to be true, but of course we cannot know that for sure. It is just a possibility, but a possibility that should be investigated, and used if possible.

A goal of this paper is also to provide the derived Bella Curve so that complicated wall geometry in buildings can be taken into account in a simple way that is easy to document.

However, if it is possible to go ahead using the two assumptions mentioned above, the simple beam theory of the Navier equation should of course not be used as basis for technical documentation of safety towards wind.

If the owners want to follow the approach of linear elasticity, the right way to do this is to formulate a finite element model (FEM) including both buildings that are present for all the twin high-rise buildings in Bellahøj, where the individual buildings are placed side-by-side with a stair-case construction between the two buildings so that the staircase might force the buildings to move together and in this way stabilize each other.

Also, we should point out that the wind force estimated in this paper is just a single force acting in one point to estimate the overturning moment as mentioned in the introduction and further discussed in Appendix C. However, since it is not a focus in this paper to discuss the difficult details of estimating the wind load on the considered structure, we have just used the wind speed in 60 % of the height, and not considered the many requirements in old or current standard about safety towards wind.

Since gust and local geometry might have an influence on the wind loads these issues should be studied performing structural health monitoring measurements on the building so that information about the wind forces and their dependence on the measured wind speed can be taken into account by adjusting the above mentioned FEM, so that it can be used as a digital twin for the two buildings connected by the staircase structure to check the total stiffness of the structure and in this way use this information to estimate the real wind force directly on the structure.

Conclusions

We have used the Navier equation to define the two dimensionless quantities i) the eccentricity degree and ii) the area degree of any beam section geometry that can be described by two flanges connected by a number of webs.

This defines the corresponding dimensionless plot denoted the Bella Curve where any beam section of the above-mentioned nature defines a simple point from where the maximum horizontal force can be easily calculated.

The corresponding wind force can then be directly compared with the maximum horizontal load making it possible to perform an estimate of the safety simply defined as the ratio between the characteristic values of the load bearing capacity and the actual wind load.

The principle is illustrated on the 10 Bellahøj buildings that might be teared down later maybe in 2028 or 29 due to lack of safety and on the considered old masonry tower in Firenze.

In both cases we find a ratio between the maximum force and the wind force of around 3.5 to 4 meaning that if the method can be used as defined in the paper, then we have lots of safety because under normal condition, the safety ratio should just be larger than 2.

For the considered Bellahøj building we have assumed that the facade elements are cast together with the rest of the building so that the facade elements and the remaining building constitute one monolithic structure which is the fundamental condition for applying the elasticity theory.

Therefore, the results of this paper do not prove safety for the Bellahøj buildings, but leads to the conclusion that the two following questions should be further investigated i) is the facade elements cast together with the rest of the building to form one monolithic structure?, and ii) is it acceptable to apply elasticity theory on the building using the total wall geometry?

These two questions should be carefully investigated before any decisions about tearing down the buildings are made.

References

[1] DS 411: Normer for Bygningskonstruktioner, Beton- og jernbetonkonstruktioner. Dansk Ingeniørforening 1949.

[2] Esben Byskov: Elementær Teori for Plane Bjælker. Department of Civil Engineering, Aalborg University. Institut for Bygningsteknik, Aalborg Universitet Nr. U0503, 2005.

[3] R.C. Ribbeler : Statics and Mechanics of Materials. Prentice Hall 2004.

[4] Bjarne Chr. Jensen et al: Teknisk Ståbi, Akademisk Forlag, 2022, (25. udgave)

[5] Internet link: Eurocode Applied Steel Tables

[6] Statens Byggeforskningsinstitut: Bellahøjhusbygger. Statens Byggeforskingsinstitut studie nr. 15. I kommission hos teknisk forlag København 1955.

[7] Dominia AS Rådgivende Ingeniører: Bellahøj I+II Betonundersøgelser. 21.08.2012.

[8] Jørgen Nielsen Rådgivende Ingeniører: Granskning af konstruktionsundersøgelser vedrørende bygningsstabilitet. 12. juli 2021.

[9] Jens-Peter Madsen ApS Rådgivende Ingeniører: Bygningsstabilitet for SAB I og SAB II Granskningsnotat. 24. April 2023.

[10] Lacanna et al: Seismic wave dispersion in high-rise historical building by interferometric analysis: The case history of Giotto’s Bell Tower. Open Access: www.taylorfrancis.com

[11] Eurocode 2: Design of concrete structures - Part 1-1 : General rules and rules for buildings. Note: See section 11, Eq. (11.2).

[12] DSEN 1991-1-4 DK NA2024: Nationalt anneks til Eurocode 1: Last på bygværker – Del 1-4: Generelle laster – Vindlast. Social- og Boligstyrelsen, 2024.

Appendix A - Review quotes

Two important reviews have been carried out in order to evaluate the structural considerations made by the administrator KAB and its consultants in order to figure out to what degree the Buildings can carry a wind load and how they should be renovated.

These two review reports are going through a lot of different structural issues that we will not consider here, but we will consider two important remarks – one from each of the two reviews – in this appendix.

First, we will consider a remark from Consulting Engineer Jørgen Nielsen's review report from 2021, see [8], page 5 above, where we give the verbatim in Danish, quote 1:

> *På SAB II er der udstøbt mellem facadeelementer og underliggende etagedæk, hvor der i projektet var regnet med ikke udstøbte fuger. Det giver en mulighed for optagelse af skivekræfter i facadeelementerne, hvilket kan have været til gunst for det stabiliserende system.*

If we translate the quote into English, we could arrive at something *like "At SAB II, there is a cast concrete joint between the facade elements and the underlying floor deck, where the project had planned for uncast joints. This provides an opportunity for the absorption of in-plane forces in the facade elements, which may have been beneficial for the stabilizing system*"

The important word here is the professional Danish term "*udstøbt"* which has a specific meaning namely that "floating concrete has been poured between the facade elements and the underlying floor slab", which again means that care has been taken to ensure a good quality of the structural function of the joint. Or a little shorter English formulation with the same technical meaning could be *"In-situ concrete has been cast between the precast facade panels and the underlying floor slab".*

However, the conclusion that is made, namely that "*horizontal forces in the facade elements might be beneficial for the stabilizing system*" as Jørgen Nielsen is mentioning, means of course that both shear forces and compression forces can be carried by the pre-cast facade elements because it works as a part of the whole monolithic structure.

Secondly, we will consider a remark from Consulting Engineer Jens Peter Madsen that he writes in his review report from 2023, see [9], page 5 above, where we have translated his remark into english, quote 2:

> *The original calculations were performed as an elastic model, where the moment of inertia was calculated on the total wall figure.*

Jens Peter Madsen is here referring to the original static calculations from 1955 that was accepted by the authorities as a proof for safety against wind and other loads.

The important thing is here, that he is saying two things, first that linear a elasticity theory is used, and next, that "*the moment of inertia was calculated on the total wall figure",* which means that the facade elements was taken into account to carry compression loads due to the wind forces and other loads.

Finally, we can conclude the following from these to quotes

A. We have well-constructed structural joints between the pre-cast facade elements and the remaining structure
B. Shear forces and compression forces can be carried by the pre-cast facade elements
C. The original calculations to prove safety was based on linear elasticity theory
D. The whole wall geometry including the pre-cast facade elements was included in the original calculations

Appendix B - Bellahøj SAB II building 6 geometry

The complex geometry of the considered building can best be illustrated by showing the layout of the rooms and walls in the considered building, see Figure A1.

The transverse walls in the building are all 15 cm unreinforced concrete including the two gable walls on the left and right in the picture. If we count them, we get around 9 transverse walls including the two gable walls which means that we have 8 sections in the building, and since for the Bellahøj SAB II buildings, the modular grid of architecture is 3.5 m in average that gives us a building length of 8 x 3.5 = 28 m which is the actual length of the considered building.

We see clearly on this plot, how difficult it would be to take all the different small details of geometry into account including the five balconies on the building and the many doors, windows and other architecturally important details into account. However, it is important to consider all to reach an accurate determination of the moment of inertia that we need when we like to use a linear elastic model for the building.

Therefore, the considered building is an excellent case to consider when we want to illustrate the value of simplicity when using the Bella Curve where we only need two quantities, where the main parameter is the percentage area of load bearing transverse walls of the building (area degree), and the secondary parameter is the relative thickness of the flange.

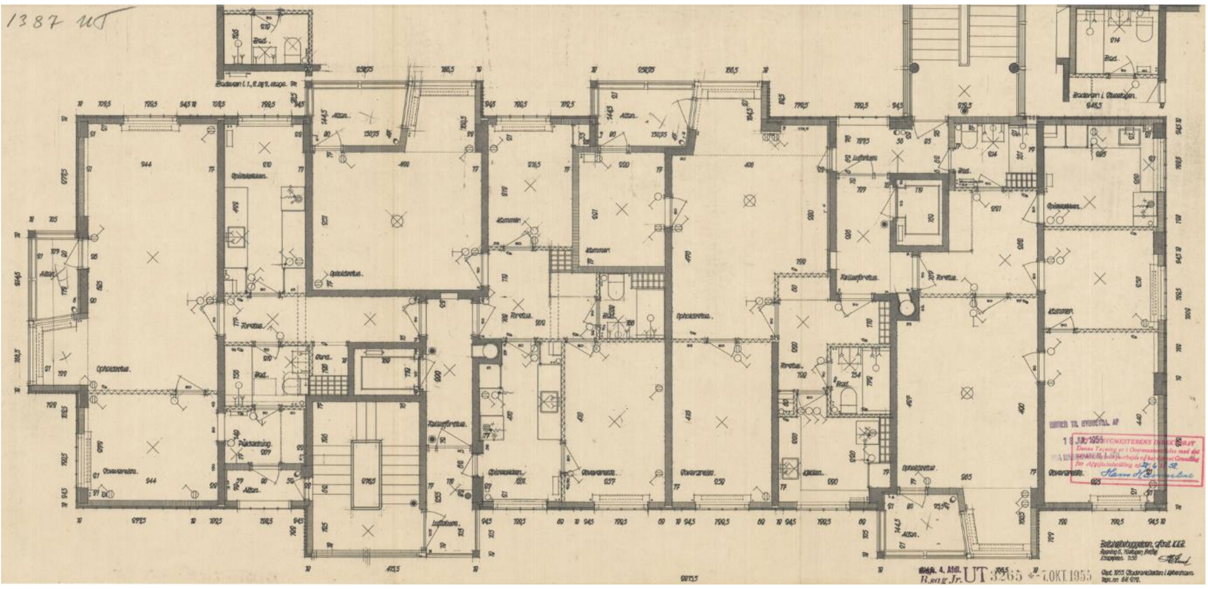

Figure A1. Layout of rooms and walls in Bellahøj SAB II building 6. Length of the building 28 m, and depth 10 measured from corner to corner.

In this case we will define the flange thickness as the average thickness of the concrete in the facades at the top and the bottom of the picture of Figure 1, and in order to do that we need to consider the geometry. We have already estimated the average length of the elements which is determined by the modular grid of architecture for Bellahøj II which is 3.5 m in average.

The thickness of the concrete of the elements is 20 cm as it is mentioned the report from 2012, see [7] where we give the verbatim in Danish, quote:

> *Facaderne er udført af præfabrikerede helvægselementer. Facadeelementerne er opbygget af en ca. 40 mm tyk betonskal med armerede ribber i tykkelsen ca. 130 mm. Betonskallerne er påstøbt lecabeton og indvendig pudset til en samlet tykkelse på ca 210 mm.*

The english translation could be like: "*The facades are constructed using prefabricated full-wall elements. The facade elements consist of a concrete shell approximately 40 mm thick with reinforced ribs approximately 130 mm thick. Lightweight aggregate concrete (Leca) is cast onto the concrete shells, and the interior is plastered to achieve a total thickness of approximately 210 mm*."

We will assume that the interior plastering is 10 mm, so that the precast elements are 200 mm when mounted, and further we will assume that the ribs are added to replace strength and stiffness for concrete shell due to the windows, which means that we will not take windows into account.

Further, we will assume two cases that will define two different estimates of the average concrete thickness:

A. The length of the elements are mounted between 30 cm width columns in the facade at the ends of the un-reinforced transversed wall and the facade elements has the combined 40 mm concrete and 160 mm leca-concrete all the way between the columns, thus the average length of the element is 3.2 m
B. The elements are like described in A, but has a 15 cm width frame at each end so that the combined 40 mm concrete and 160 mm leca-concrete is reduced to 2.9 m.

Since the leca concrete has a weight of around 600 kg/m3, the stiffness of the leca concrete is reduced by a factor 15, see [11][5] and we get the following effective concrete thickness over a modular length for the two cases

A. Concrete area = 4 cm, effective thickness from leca-conctrete: 16/15 = 1.1 cm and from the columns: 20 x 30/320 = 1.9 cm, so that the total effective thickness is 4 + 1.1 + 1.9 = 7.0 cm.

[5] See section 11.3.2

B. The same as A, but now from the columns: 20 x 60/290 = 1.9 = 4.1 cm, so that the total effective thickness is 4 + 1.1 + 4,1 = 9.2 cm.

Appendix C - Wind forces

In this appendix the focus is on providing an estimate of the wind forces on the two considered buildings that we can argue is a reasonable estimate of a characteristic value in the classical sense of partial coefficients for safety, where safety is secured when the ratio between the wind force and the load bearing capacity of the building is minimum a factor 2.

We will not use current or old standards for the wind loads, but only rely on well accepted principles and physical laws in order to reach the wind force estimates.

In Denmark the reference wind v_{ref} (also denoted the basis wind) is defined as the wind speed in 10 m height and is in the most of Denmark set to $v_{ref} = 24$ m/s, see [12], which is the value that we will use here. However, we need to scale the wind speed to the right value for the building, and since we have chosen to look at only one single force, we will estimate the wind speed in this height. In order to be on the safe side using an upper bound for the wind force we will use the generally accepted rule of thumb of considering the single force in 60 % of the height h_{60} = 24 m, and we will also calculate the wind force in this height.

In order to scale wind speed $v(z)$ to an arbitrary height z we will use the formula for the logarithmic wind profile

(C.1) $$v(z) = \frac{v_*}{\kappa} \ln(z / z_0)$$

where v_* is the so-called friction velocity, κ is the Von Kármán constant equal to 0.40, and z_0 is the roughness length of the terrain. For Bellahøj we will use the roughness length according to suburb area equal to $z_0 = 0.35$ m, and then if we use v_{ref} and the corresponding height in this equation, we can then find the friction velocity to $v_* = 2{,}9$ m/s.

This leads to a static wind speed at 60 % of the heigh that is $v_s = 30{,}3$ m/s which is the wind speed we will use to find the total static wind force F_{vs} on the building using the drag equation

(C.2) $$F_{vs} = \frac{1}{2} \rho v_s^2 C A$$

where ρ is the mass density of the air, ρ = 1.25 kg/m3, C is the shape factor that for a normal box-shaped building is C = 1.3 which includes both the wind pressure on the windward side and the wind suction on the leeward side. A is the areas of the facade, A = 40 x 28 = 1120 m2, and we find F_{vs} = 0.84 MN corresponding to 84 tons.

However, this wind speed is just the mean value, and we have also to know the random contribution in order to calculate a reasonable wind force including the peak loads. For the random wind force we will use that the standard deviation σ_v of the random wind speed is proportional to the friction velocity according to the simple empirical formula

(C.3) $$\sigma_v = 2.5 \times v_*$$

which gives us the value $\sigma_v = 7.2$ m/s. In principle, when we calculate the wind force, we should integrate the moment contributions over the facade area to find the overturning moment

(C.4) $$M(t) = \int_A q(t,z) z dA$$

Where $q(t,z)$ is the wind pressure including also random contributions and thus is depending on time and height. When the moment $M(t)$ has been estimated, then we can obtain the wind force at 60 % of the height as $F_v(t) = M(t)/h_{60}$ .However, we will not do it this way, but follow more simple principles.

Since in principle we integrate over the area to estimate a single force then a lot of independent contributions are averaged together, and as result, the standard deviation on the single force is reduced accordingly. We will assume that we have in this case approximately 20 independent contributions, and thus the standard deviation is divided by a factor $\sqrt{20}$.

When we have added many stochastic contributions together, we end up with an averaged result that is normally distributed according to the central limit theorem, and we then reach a 10th percentile by multiplying the standard deviation by a factor 1.3. Thus, we will estimate the characteristic value of the wind speed peak as

(C.5) $$v_p = v_s + 1.3 \times \sigma_v / \sqrt{20} = v_s(1+e)$$

where $e = 1.3 \times \sigma_v / (v_s \sqrt{20})$ gives us the wind speed peak factor $(1+e)$ = 1.069. This means that since we are squaring the velocity in order to find the force according to Eq (A.2), we can find the similar peak value for the force as

(C.6) $$F_{vp} = F_{vs}(1+2e)$$

So we have a wind force peak factor of $(1+2e)$ = 1.14. So, we just add 14 % to the static force and arrive at the peak value of the wind force F_{vp} = 95 tons for the Bellahøj building.

The main differences between Bellahøj and Giotto Tower is that both the terrain roughness and the height is considerable larger for the Giotto Tower case. That influence as explained above the standard deviation of the wind speed that for the Giotto tower is now increased to $\sigma_v = 12.7$ m/s. Further, it is evaluated that the number of independent stochastic contributions is reduced to a factor 10 due to the slender geometry, and as a result the standard deviation is divided only by a factor $\sqrt{10}$, but due to larger static wind speed v_s = 44.6 m/s the force peak is now reduced to a factor $(1+2e)$ = 1.08. Final result is that the larger height and the larger wind speed result in a value of the peak value of the wind force F_{vp} = 246 tons for the Giotto Tower.

Table C.1. *Key wind parameters for determination of wind forces.*

The two Buildings	**Terrain roughness [m]**	**60 % of height [m]**	**Static wind speed [m/s]**	**Static force [tons]**	**Force peak [tons]**
Bellahøj	0,35	24,00	30,27	83	95
Giotto Tower	1,50	51,00	44,61	199	246